The AdS/CFT Correspondence for the Massive Rarita-Schwinger Field

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Abstract

The complete solution to the massive Rarita-Schwinger field equation in anti-de Sitter space is constructed, and used in the AdS/CFT correspondence to calculate the correlators for the boundary conformal field theory. It is found that when no condition is imposed on the field solution, there appear two different boundary conformal field operators, one coupling to a Rarita-Schwinger field and the other to a Dirac field. These two operators are seen to have different scaling dimensions, with that of the spinor-coupled operator exhibiting non-analytic mass dependence.

1 Introduction

The Maldacena conjecture [1] asserts that there exists a holographic correspondence [2] between field theories on $d+1$-dimensional AdS space and conformal field theories on the d-dimensional boundary of this space. This correspondence has been made more precise in [1, 3, 4, 5, 6] and investigated for specific cases in [7]-[17] We do not attempt to give a comprehensive list of references here, but refer the reader to the literature. A recent review of the Maldacena conjecture can be found in [18]. According to this correspondence principle, the action for the field theory in the bulk AdS space written in terms of the boundary values of the fields serves as a generating functional for a field theory which lives on the flat boundary space. This can be written

$$Z_{AdS}[\psi_{(0)}] = \int_{\psi_{(0)}} \mathcal{D}\psi e^{-I[\psi]} = Z_{CFT}[\psi_{(0)}] = \left\langle e^{\int_{\partial \mathrm{AdS}} \mathrm{d}^d x \mathcal{O}\psi_{(0)}} \right\rangle . \tag{1}$$

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where $\psi_{(0)}$ is the boundary field, and acts as a source for the operator $\mathcal{O}$. Since we deal in the present case with a classical field, we obtain an approximation to the path integral.[1]

We will choose the AdS metric to be $g_{\mu\nu} = \frac{1}{x^{0\,2}}\delta_{\mu\nu}$ so that the boundary is at $x^0 = 0$. Since the metric diverges on this boundary, we must regularise by multiplying by a function with a suitable zero on the boundary. [5] The fact that this function is otherwise unspecified is the origin of the conformal invariance in the boundary field theory.

Of particular interest here are [12, 13, 16, 17] which also deal with the Rarita-Schwinger field, but impose restrictions on the solution of the field equation. We find that when the general solution is used, there appear two fields on the d-dimensional boundary. Both a Dirac spinor and a spin 3/2 Rarita-Schwinger field couple to boundary conformal field operators, which as a result have different conformal scaling dimensions. To find these conformal field correlators, we use the Dirichlet boundary value problem method exhibited in, for example, [9] for the case of a Dirac spinor field. Since the action vanishes on-shell, a surface term must be added.[2] Two equivalent methods [19, 20] of determining this term have been investigated, and the method of [20] has recently been used in [17].

In following this prescription, we solve the equations of motion in section 2. The surface term to add to the action is found using the method of [20] in section 3, and finally in section 4 the CFT correlators are calculated. These correlators are fixed, up to a multiplicative factor, by conformal invariance [21, 22]. The results obtained are consistent with these considerations.

2 Solving the Classical field Equations

Although the equations of motion have been solved in [12] for the massless case, in [13] for the case of $\gamma^\mu \psi_\mu = 0$, and somewhat more generally in [16], we find it necessary to construct explicitly the complete solution to the massive[3] case while imposing no restrictions.

Our index conventions are $\mu, \nu, ... = 0...d$ and $i, j, ... = 1...d$. We choose the metric of AdS_{d+1} to be $g_{\mu\nu} = \frac{1}{x^{0\,2}}\delta_{\mu\nu}$ so that AdS space is given by $x^0 > 0$. The boundary with which we shall be concerned is at $x^0 = 0$, where the metric is singular. The Rarita-Schwinger

[1] An investigation of the AdS/CFT correspondence which deals with quantum corrections in given in [10]

[2] The equations of motion, of course, do not change.

[3] As pointed out in [16], one must consider $m_1 \neq 0$ in the case of supergravity on $AdS_5 \times S^5$ [23].

action is given by

$$I = \int d^{d+1}x\sqrt{g}\overline{\psi}_\mu[\Gamma^{\mu\nu\sigma}D_\nu - m_1 g^{\mu\sigma} - m_2\Gamma^{\mu\sigma}]\psi_\sigma. \tag{2}$$

D_ν denotes the covariant derivative, Γ_μ are curved space Dirac matrices so that $\Gamma_\mu = e_\mu^a\gamma_a$ where $\{\gamma_\mu,\gamma_\nu\} = 2\delta_{\mu\nu}$ are Euclidean Dirac matrices, which are taken to be Hermitian, and the vielbein is given by $e_\mu^a = \frac{1}{x^0}\delta_\mu^a$. More than one index on these matrices indicates antisymmetrisation (including $1/n!$). Varying the above action gives the Rarita-Schwinger equation

$$[\Gamma^{\mu\nu\sigma}D_\nu - m_1 g^{\mu\sigma} - m_2\Gamma^{\mu\sigma}]\psi_\sigma = 0. \tag{3}$$

and its conjugate

$$\overline{\psi}_\mu[\Gamma^{\mu\nu\sigma}\overleftarrow{D}_\nu + m_1 g^{\mu\sigma} + m_2\Gamma^{\mu\sigma}] = 0. \tag{4}$$

we will find it convenient to write the former in the equivalent form

$$\Gamma^\nu[D_\nu\psi_\mu - D_\mu\psi_\nu] + \frac{m_+}{d-1}\Gamma_\mu\Gamma^\nu\psi_\nu - m_-\psi_\mu = 0 \tag{5}$$

which can be seen by using $\Gamma^{\mu\nu\sigma} = \frac{1}{2}(\Gamma^\nu\Gamma^\sigma\Gamma^\mu - \Gamma^\mu\Gamma^\sigma\Gamma^\nu)$. To solve this equation generally, we first contract with D_μ to obtain:

$$\Gamma^{\mu\nu\sigma}[D_\nu, D_\mu]\psi_\sigma - 2m_1 D^\mu\psi_\mu - m_2\Gamma^{\mu\nu}D_{[\mu}\psi_{\nu]} = 0. \tag{6}$$

The commutator $[D_\nu, D_\mu]\psi_\sigma$ can be expressed as:

$$[D_\mu, D_\nu]\psi_\sigma = \left(\frac{1}{2}\partial_{[\mu}\omega_{\nu]} + \frac{1}{4}[\omega_\mu,\omega_\nu]\right)\psi_\sigma = \frac{1}{2}R_{\mu\nu}\psi_\sigma \tag{7}$$

where the spin connection is given by

$$\omega_\mu^{AB} = \frac{1}{x^0}(\delta_0^A\delta_\mu^B - \delta_0^B\delta_\mu^A) \tag{8}$$

and $\omega_\mu = \omega_\mu^{AB}\Sigma_{AB}$. The computation of $R_{\mu\nu}$ is simplified if we make use of the fact that the space is maximally symmetric [24]. We find

$$R_{\mu\nu} = \frac{R}{d(d+1)}[\Gamma_\mu,\Gamma_\nu] \tag{9}$$

and in our metric $R = -d/(d+1)$ so that

$$\Gamma^{\mu\nu\sigma}[D_\nu, D_\mu]\psi_\sigma = \frac{d(1-d)}{2}\Gamma^\sigma\psi_\sigma \tag{10}$$

and (6) becomes

$$m_2 \not{D}(\Gamma^\mu\psi_\mu) + m_- D^\mu\psi_\mu + \frac{d(d-1)}{4}\Gamma^\mu\psi_\mu = 0 \tag{11}$$

Now we contract (3) with Γ_μ. Using $\Gamma_\mu\Gamma^{\mu\nu\sigma} = (d-1)\Gamma^{\mu\nu}$ we find

$$\not{D}(\Gamma^\mu\psi_\mu) - D^\mu\psi_\mu + \frac{m_1 + dm_2}{1-d}\Gamma^\mu\psi_\mu = 0. \tag{12}$$

Combining (11) and (12) to eliminate $D^\mu\psi_\mu$, we obtain a Dirac equation

$$[\not{D} - C](\Gamma^\nu\psi_\nu) = 0 \tag{13}$$

where

$$C = \frac{d(d-1)}{4m_1} + \frac{(m_1 + dm_2)m_-}{m_1(d-1)} \tag{14}$$

and for convenience we have defined $m_\pm = m_1 \pm m_2$. It can also be shown from (11) and (12) that $m_1 = 0$ implies $\gamma^\mu\psi_\mu = 0$. Since this case has been considered in [12] and [13], we assume $m_1 \neq 0$.

Now we specialise to our coordinate system and write (13) as

$$\left(x^0\not{\partial} - \frac{d}{2}\gamma_0 - C\right)\gamma\cdot\psi = 0 \tag{15}$$

where we will now work only with the vielbein components of ψ.

This equation has been solved in [9] by differentiating to obtain a second-order equation, and has the solution which does not diverge as $x^0 \to \infty$

$$\gamma\cdot\psi = (kx^0)^{\frac{d+1}{2}}\left[A^{(1)}K_{C+\frac{1}{2}}(kx^0) + A^{(3)}K_{C-\frac{1}{2}}(kx^0)\right] \tag{16}$$

where $A^{(1)}$ and $A^{(3)}$ are spinors which do not depend on x^0. Since this form of the solution was found via a second order equation, equation (16) needs to be substituted back into the first-order equation (15) in order to find these spinors. We write $x = (x^0, \mathbf{x})$, and we will work in Fourier space with respect to the non-zero-index components of the field

$$\tilde{\psi}_\mu(x^0, \mathbf{k}) = \int d^d x e^{i\mathbf{k}\cdot\mathbf{x}}\psi_\mu(x) \tag{17}$$

Since we will soon need to work with several other first-order equations and doing the full calculation every time would be tedious, we calculate the following formula

$$\begin{aligned}
&\left[x^0\gamma_0\partial_0 - ix^0\mathbf{k}\cdot\boldsymbol{\gamma} - n\gamma_0 - P\right](kx^0)^l\left[\big(A^{(1)}+(kx^0)A^{(2)}\big)K_{q+\frac{1}{2}} + \big(A^{(3)}+(kx^0)A^{(4)}\big)K_{q-\frac{1}{2}}\right]\\
&= (kx^0)^l\Bigg\{K_{q+\frac{1}{2}}\left[(l-n-q-\frac{1}{2})\gamma_0 - P\right]A^{(1)} + K_{q-\frac{1}{2}}\left[(l-n+q-\frac{1}{2})\gamma_0 - P\right]A^{(3)}\\
&\quad + (kx^0)K_{q+\frac{1}{2}}\left[-i\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k}A^{(1)} + \big((l-n-q+\frac{1}{2})\gamma_0 - P\big)A^{(2)} - \gamma_0 A^{(3)}\right]\\
&\quad + (kx^0)K_{q-\frac{1}{2}}\left[-i\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k}A^{(3)} + \big((l-n+q+\frac{1}{2})\gamma_0 - P\big)A^{(4)} - \gamma_0 A^{(1)}\right]\\
&\quad + (kx^0)^2K_{q+\frac{1}{2}}\left[-i\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k}A^{(2)} - \gamma_0 A^{(4)}\right] + (kx^0)^2K_{q-\frac{1}{2}}\left[-i\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k}A^{(4)} - \gamma_0 A^{(2)}\right]\Bigg\} \qquad (18)
\end{aligned}$$

where P, l, n, q are arbitrary constants, the A are spinors which may depend on $\mathbf{k}$, and from now on we omit the arguments (kx^0) of the Bessel functions. Our present case of $\gamma\cdot\tilde{\psi}$ corresponds to (18) with $A^{(2)} = A^{(4)} = 0$ and $q = P = C, n = \frac{d}{2}, l = \frac{d+1}{2}$. Requiring the resulting RHS of (18) to vanish gives $A^{(1)}$ and $A^{(3)}$, so that, writing $k = |\mathbf{k}|$,

$$\gamma\cdot\tilde{\psi} = (kx^0)^{\frac{d+1}{2}}\left[i\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k}K_{C+\frac{1}{2}} + K_{C-\frac{1}{2}}\right]b_0^+(\mathbf{k}), \qquad (19)$$

where b_0^+ is a free spinor function of $\mathbf{k}$. (We will consistently use $+$ and $-$ superscripts to denote eigenspinors of γ_0 with eigenvalues $+1$ and -1.)

The equation of motion (3) in coordinate component form is

$$\left[x^0\gamma\cdot\partial - \frac{d}{2}\gamma_0 - m_-\right]\psi_a + \left[\frac{3}{2}\delta_{a0} - x^0\partial_a - \frac{1}{2}\gamma_0\gamma_a + \frac{m_+}{d-1}\gamma_a\right]\gamma\cdot\psi = \gamma_a\psi_0. \qquad (20)$$

The $a = 0$ component of (20) is

$$\left[x^0\gamma_0\partial_0 - ix^0\mathbf{k}\cdot\boldsymbol{\gamma} - (\frac{d}{2}+1)\gamma_0 - m_-\right]\tilde{\psi}_0 = \left(x^0\partial_0 - 1 - \frac{m_+}{d-1}\gamma_0\right)\gamma\cdot\tilde{\psi} \qquad (21)$$

We will find both a particular and homogeneous solution for (21). Since we already know the RHS, we make the ansatz

$$\tilde{\psi}_0^{\,P} = (kx^0)^{\frac{d+1}{2}}\left[\big(A^{(1)}+(kx^0)A^{(2)}\big)K_{C+\frac{1}{2}} + \big(A^{(3)}+(kx^0)A^{(4)}\big)K_{C-\frac{1}{2}}\right]. \qquad (22)$$

(Note that we re-use the parameters $A^{(1)}, A^{(2)}, A^{(3)}, A^{(4)}$ each time we do a calculation with (18).) Reading off the LHS of (21) as the RHS from (18) with $P = m_-, q = C, n = \frac{d}{2}+1, l = \frac{d+1}{2}$ and matching the coefficients of the linearly independent functions of kx^0 with those on the RHS of (21), we obtain equations which can be solved for the A parameters. The result is

$$\tilde{\psi_0}^P = (kx^0)^{\frac{d+1}{2}} \left\{ K_{C+\frac{1}{2}} \left[i\mu_2 \frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k} + \mu_3(kx^0) \right] b_0^+ + K_{C-\frac{1}{2}} \left[-\mu_1 + i\mu_3(kx^0)\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k} \right] b_0^+ \right\}, \tag{23}$$

where

$$\mu_1 = \frac{\frac{m_+}{d-1} - C - \frac{d}{2} + 1}{C - m_- - 1} \qquad \mu_2 = \frac{\frac{m_+}{d-1} - C + \frac{d}{2} - 1}{C - m_- + 1} \qquad \mu_3 = \frac{1 + \mu_1 + \mu_2}{C + m_-} \tag{24}$$

Useful relations involving these constants are contained in the appendix.

The homogeneous version of (21) is exactly the same as (15) but with $\frac{d}{2}$ and C replaced by $\frac{d}{2}+1$ and m_-, respectively. Therefore our solution is (19) with the same changes:

$$\tilde{\psi_0}^H = (kx^0)^{\frac{d+3}{2}} \left[i\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k} K_{m_-+\frac{1}{2}} + K_{m_--\frac{1}{2}} \right] c_0^+(\mathbf{k}). \tag{25}$$

The complete solution for $\tilde{\psi}_0$ is just the sum of the two parts (23) and (25).

To find $\tilde{\psi}_i$ we use the $a = i$ components of (20):

$$\left[x^0\gamma_0\partial_0 - ix^0\mathbf{k}\cdot\boldsymbol{\gamma} - \frac{d}{2}\gamma_0 - m_- \right] \tilde{\psi}_i = \gamma_i\tilde{\psi}_0 + \left[\frac{1}{2}\gamma_0\gamma_i - ix^0 k_i - \frac{m_+}{d-1}\gamma_i \right] \gamma\cdot\tilde{\psi} \tag{26}$$

On the RHS of (26), we have terms from (23), (25) and (19). These terms all consist of some power of kx^0 and a Bessel function of order $C \pm \frac{1}{2}$ or $m_- \pm \frac{1}{2}$. We consider $\tilde{\psi}_i$ in three parts: $\tilde{\psi}_i = \tilde{\psi_i}^H + \tilde{\psi_i}^C + \tilde{\psi_i}^{m_-}$. The homogeneous equation is once again the same as (15) with the replacement $C \to m_-$, so we have

$$\tilde{\psi_i}^H = (kx^0)^{\frac{d+1}{2}} \left[i\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k} K_{m_-+\frac{1}{2}} + K_{m_--\frac{1}{2}} \right] b_i^+(\mathbf{k}). \tag{27}$$

where $b_i^-(\mathbf{k})$ is free. Re-using our A parameters in (18), we make the ansatz

$$\tilde{\psi_i}^C = (kx^0)^{\frac{d+1}{2}} \left[\left(A_i^{(1)} + (kx^0)A_i^{(2)} \right) K_{C+\frac{1}{2}} + (A_i^{(3)} + (kx^0)A_i^{(4)}) K_{C-\frac{1}{2}} \right]. \tag{28}$$

Evaluating (18) with $P = m_-, q = C, n = \frac{d}{2}, l = \frac{d+1}{2}$ and matching the result with the corresponding Bessel functions on the RHS of (26) we obtain five equations for the four parameters, but they are consistent, and the result is

$$\begin{aligned}\tilde{\psi}_i^{\,C} &= (kx^0)^{\frac{d+1}{2}}\Bigg\{K_{C+\frac{1}{2}}\left[-i\frac{\mu_2 - \frac{m_+}{d-1} + \frac{1}{2}}{C+m_-}\gamma_i - (kx^0)\mu_3\frac{k_i}{k}\right]\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k}b_0^+ \\ &\quad + K_{C-\frac{1}{2}}\left[\frac{\mu_1 + \frac{m_+}{d-1} + \frac{1}{2}}{C+m_-}\gamma_i + i(kx^0)\mu_3\frac{k_i}{k}\right]b_0^+\Bigg\} \qquad (29)\end{aligned}$$

Exactly the same procedure, using the ansatz

$$\tilde{\psi}_i^{\,m_-} = (kx^0)^{\frac{d+1}{2}}\left[\left(A_i^{(1)} + (kx^0)A_i^{(2)}\right)K_{m_-+\frac{1}{2}} + \left(A_i^{(3)} + (kx^0)A_i^{(4)}\right)K_{m_--\frac{1}{2}}\right], \qquad (30)$$

yields the solution

$$\begin{aligned}\tilde{\psi}_i^{\,m_-} &= (kx^0)^{\frac{d+1}{2}}\Bigg\{K_{m_-+\frac{1}{2}}\left[i\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k}c_i^+(\mathbf{k}) - (2m_-+1)\frac{k_i}{k}\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k}c_0^+ + \gamma_i c_0^+ + i(kx^0)\frac{k_i}{k}c_0^+\right] \\ &\quad + K_{m_--\frac{1}{2}}\left[c_i^+(\mathbf{k}) - (kx^0)\frac{k_i}{k}\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k}c_0^+\right]\Bigg\} \qquad (31)\end{aligned}$$

where $c_i^+(\mathbf{k})$ is free.

At this point we notice that when (27), (29), and (31) are combined, the free quantities c_i^+ and b_i^+ always appear together as $c_i^+ + b_i^+$; we thus lose no generality in choosing $c_i^+ = 0$.

Lastly, for the entire solution to be consistent, we require that $\gamma_0\tilde{\psi}_0 + \gamma_i\tilde{\psi}_i$ calculated from (23)+(25) and (27)+(29)+(31) be equal to the same quantity $\gamma\cdot\tilde{\psi}$ given by (19). Equating the two gives a formula

$$c_0^+ = i\frac{1+\mu_1}{m_1}\frac{\mathbf{k}\cdot\mathbf{b}^+}{k} \qquad (32)$$

and also a condition on the otherwise free b_i^+

$$\boldsymbol{\gamma}\cdot\mathbf{b}^+ = 0 \qquad (33)$$

The complete solution for the field $\tilde{\psi}_a$ is thus given by (23) and (25),

$$\begin{aligned}\tilde{\psi}_0 &= (kx^0)^{\frac{d+1}{2}}\Bigg\{K_{C+\frac{1}{2}}\left[i\mu_2\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k} + (kx^0)\mu_3\right]b_0^+ + K_{C-\frac{1}{2}}\left[-\mu_1 + i(kx^0)\mu_3\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k}\right]b_0^+ \\ &\quad + kx^0\left[-\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k}K_{m_-+\frac{1}{2}} + iK_{m_--\frac{1}{2}}\right]\frac{1+\mu_1}{m_1}\frac{\mathbf{k}\cdot\mathbf{b}^+}{k}\Bigg\} \qquad (34)\end{aligned}$$

and by (27), (29), and (31):

$$
\begin{aligned}
\tilde{\psi}_i = (kx^0)^{\frac{d+1}{2}} \Bigg\{ & K_{C+\frac{1}{2}} \left[i\frac{1+\mu_2}{d}\gamma_i - (kx^0)\mu_3 \frac{k_i}{k} \right] \frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k} b_0^+ \\
& + K_{C-\frac{1}{2}} \left[\frac{1+\mu_1}{d}\gamma_i + i(kx^0)\mu_3 \frac{k_i}{k} \right] b_0^+ \\
& + K_{m_-+\frac{1}{2}} \left[i\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k} b_i^+ + \left(i\gamma_i - i(2m_- + 1)\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k}\frac{k_i}{k} - (kx^0)\frac{k_i}{k} \right) \frac{1+\mu_1}{m_1}\frac{\mathbf{k}\cdot\mathbf{b}^+}{k} \right] \\
& + K_{m_--\frac{1}{2}} \left[b_i^+ - i(kx^0)\frac{1+\mu_1}{m_1}\frac{k_i}{k}\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k}\frac{\mathbf{k}\cdot\mathbf{b}^+}{k} \right] \Bigg\}
\end{aligned} \tag{35}
$$

where $b_0^+(\mathbf{k})$ is free, and $b_i^+(\mathbf{k})$ are subject only to (33). It can be seen that the solutions found in [12], [13], and [16] are special cases of the above solution.

The form of the solution to the conjugate equation (4) may be found by conjugating this result. Care must be taken when defining the 'bar' operation. We define it in the following way,

$$\overline{X} \equiv X^\dagger \big|_{m \to -m} \tag{36}$$

where $m \to -m$ means that we change the sign of both m_1 and m_2. Under this operation, $m_- \to -m_-$, $m_+ \to -m_+$, $C \to -C$, $\mu_1 \leftrightarrow \mu_2$, and $\mu_3 \to -\mu_3$. It is of special importance to realise that ψ and $\overline{\psi}$ are independent quantities; taking the conjugate of ψ in this way only gives the form of the solution for $\overline{\psi}$, and the resulting arbitrary functions $\overline{b}_i(\mathbf{k})$ and $\overline{b}_0(\mathbf{k})$ will be unrelated to b_i and b_0. In all other cases, the operation of conjugation will produce not independent quantities, but conjugated ones. We will make frequent use of this notation in the remainder of this paper.

3 Adding a Surface Term to the Action

Along the lines of [20] we vary the action (2) and examine surface terms which do not vanish when the equations of motion (3), (4) are satisfied. This has been done in [17] and the result is that the term we need to add to the action is, in the notation of [20],

$$C_\infty = \frac{1}{2} \int \mathrm{d}^d x \sqrt{h} \left(\overline{\psi}_{i(0)}\psi_{i(0)} + \overline{\psi}_{i(0)}\gamma_i\gamma_j\psi_{j(0)} \right) \tag{37}$$

where h is the induced metric on the boundary.[4] Thus when we insert the classical solution into the action, only this surface term remains and it can be written in the form

$$I = \frac{1}{2}\int \mathrm{d}^d x \mathrm{d}^d y \sqrt{h} \int \frac{\mathrm{d}^d k}{(2\pi)^d} e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} \left(\overline{\psi}_{i(0)}(\mathbf{k})\psi_{i(0)}(-\mathbf{k}) + \overline{\psi}_{i(0)}(\mathbf{k})\gamma_i\gamma_j\psi_{j(0)}(-\mathbf{k})\right) \tag{38}$$

We shall use this in the next section to calculate the correlators. This amounts to doing the k-integral in (38) and taking the $\epsilon \to 0$ limit. As observed in [9], this must be done with care by formulating a Dirichlet boundary value problem not simply at $x^0 = 0$ but at ϵ and taking the limit at the end.

The formula which will be needed for this integral[5], properly regularised [25], is

$$\int \frac{\mathrm{d}^d k}{(2\pi)^d} e^{-i\mathbf{k}\cdot\mathbf{x}} k^q = \frac{2^q \Gamma\left(\frac{d+q}{2}\right)}{\pi^{d/2}\Gamma\left(\frac{-q}{2}\right)} \frac{1}{|\mathbf{x}|^{d+q}} \tag{39}$$

It will be crucial in section 4 to realise that for q a non-negative even integer, this integral vanishes.

4 Boundary CFT correlator

In the case of a spinor field [9], it was found that as the boundary at $x^0 = \epsilon \to 0$ is approached, ψ^+ and ψ^- are related by a factor of some power of ϵ with the consequence that one may be specified on the boundary, while the other must vanish. Since we will find the same behaviour in the present case, we split the field into two parts, ψ^+ and ψ^-. We now set about inverting (35) to write the parameters b_i and b_0 in terms of the boundary fields $\psi_i(k\epsilon)$, which we will abbreviate as $\psi_{i\epsilon}$. From (35) we have

$$\begin{aligned}\psi^+_{i\epsilon} = (k\epsilon)^{\frac{d+1}{2}} \Bigg\{ i\left[K_{C+\frac{1}{2}} \frac{1+\mu_2}{d}\gamma_i \frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k} + k\epsilon K_{C-\frac{1}{2}}\mu_3 \frac{k_i}{k}\right] b_0^+ \\ + K_{m_- -\frac{1}{2}} b_i^+ - k\epsilon K_{m_- +\frac{1}{2}} \frac{1+\mu_1}{m_1} \frac{k_i}{k} \frac{\mathbf{k}\cdot\mathbf{b}^+}{k} \Bigg\}\end{aligned} \tag{40}$$

$$\begin{aligned}\psi^-_{i\epsilon} = (k\epsilon)^{\frac{d+1}{2}} \Bigg\{ \left[K_{C-\frac{1}{2}} \frac{1+\mu_1}{d}\gamma_i - k\epsilon K_{C+\frac{1}{2}}\mu_3 \frac{k_i}{k}\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k}\right] b_0^+ + iK_{m_- +\frac{1}{2}} \frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k} b_i^+ \\ + \left[K_{m_- +\frac{1}{2}}\left(\gamma_i - (2m_- + 1)\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k}\frac{k_i}{k}\right) - k\epsilon K_{m_- -\frac{1}{2}} \frac{k_i}{k}\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k}\right] i\frac{1+\mu_1}{m_1}\frac{\mathbf{k}\cdot\mathbf{b}^+}{k} \Bigg\}\end{aligned} \tag{41}$$

[4] Actually, h is the induced metric on the surface at $x^0 = \epsilon$.

[5] We will see later on that we have no need of local terms; we do not include here terms which contribute only when $\mathbf{x}$=0.

Multiplying (40) from the left by γ_i and alternately by $\frac{k_i}{k}$ gives two equations which can be solved simultaneously for b_0 and $\mathbf{k}\cdot\mathbf{b}$ in terms of $\psi_{i\epsilon}$. We note that in contrast to [16], where a similar parameter b_0 cannot be determined in terms of the boundary data and is thus set to zero, b_0 here can be written in terms of the boundary field. Substituting b_0 back into (40) now allows us to solve for b_i. Inserting these expressions for b_0, $\mathbf{k}\cdot\mathbf{b}$, and b_i into (41), $\psi^-_{i\epsilon}$ can be expressed in terms of $\psi^+_{i\epsilon}$. Since the b parameters are eliminated in this process, we are not free to impose any restrictions on them. The result is

$$\psi^-_{i\epsilon} = O_{ij}\psi^+_{j\epsilon} \tag{42}$$

where

$$O_{ij} = f_1\gamma_i\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k}\gamma_j + f_2\frac{k_i}{k}\gamma_j + f_3\gamma_i\frac{k_j}{k} + f_4\frac{k_i}{k}\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k}\frac{k_j}{k} + f_5\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k}\delta_{ij} \tag{43}$$

and the each f is a purely imaginary function of $(k\epsilon)$,

$$\begin{aligned} f_1 =& \frac{1}{D}\left[K_{C-\frac{1}{2}}K_{m_- -\frac{1}{2}}\frac{1+\mu_1}{d} - K_{C+\frac{1}{2}}K_{m_- +\frac{1}{2}}\frac{1+\mu_2}{d}\right]\left[K_{m_- -\frac{1}{2}} - k\epsilon K_{m_- +\frac{1}{2}}\frac{1+\mu_1}{m_1}\right] \\ &+ \frac{1}{D}K_{m_- +\frac{1}{2}}K_{m_- -\frac{1}{2}}\frac{1+\mu_1}{m_1}\left[K_{C+\frac{1}{2}}\frac{1+\mu_2}{d} + k\epsilon K_{C-\frac{1}{2}}\mu_3\right]\end{aligned}$$

$$\begin{aligned} f_2 =& \frac{1}{D}\left[2K_{C+\frac{1}{2}}K_{m_- +\frac{1}{2}}\frac{1+\mu_2}{d} + k\epsilon K_{C-\frac{1}{2}}K_{m_- +\frac{1}{2}}\mu_3 - k\epsilon K_{C+\frac{1}{2}}K_{m_- -\frac{1}{2}}\mu_3\right]\left[K_{m_- -\frac{1}{2}} - k\epsilon K_{m_- +\frac{1}{2}}\frac{1+\mu_1}{m_1}\right] \\ &+ \frac{1}{D}\frac{1+\mu_1}{m_1}\left[k\epsilon K^2_{m_- +\frac{1}{2}} - k\epsilon K^2_{m_- -\frac{1}{2}} - (2m_- + 1)K_{m_- +\frac{1}{2}}K_{m_- -\frac{1}{2}}\right]\left[K_{C+\frac{1}{2}}\frac{1+\mu_2}{d} + k\epsilon K_{C-\frac{1}{2}}\mu_3\right]\end{aligned}$$

$$\begin{aligned} f_3 =& \frac{1}{D}\left[K_{C-\frac{1}{2}}K_{m_- -\frac{1}{2}}\frac{1+\mu_1}{d} - K_{C+\frac{1}{2}}K_{m_- +\frac{1}{2}}\frac{1+\mu_2}{d}\right]k\epsilon K_{m_- +\frac{1}{2}}\frac{1+\mu_1}{m_1} \\ &- \frac{1}{D}K_{m_- +\frac{1}{2}}K_{m_- -\frac{1}{2}}\frac{1+\mu_1}{m_1}\left[K_{C+\frac{1}{2}}(1+\mu_2) + k\epsilon K_{C-\frac{1}{2}}\mu_3\right]\end{aligned}$$

$$\begin{aligned} f_4 =& \frac{1}{D}\left[2K_{C+\frac{1}{2}}K_{m_- +\frac{1}{2}}\frac{1+\mu_2}{d} + k\epsilon K_{C-\frac{1}{2}}K_{m_- +\frac{1}{2}}\mu_3 - k\epsilon K_{C+\frac{1}{2}}K_{m_- -\frac{1}{2}}\mu_3\right]k\epsilon K_{m_- +\frac{1}{2}}\frac{1+\mu_1}{m_1} \\ &- \frac{1}{D}\frac{1+\mu_1}{m_1}\left[k\epsilon K^2_{m_- +\frac{1}{2}} - k\epsilon K^2_{m_- -\frac{1}{2}} - (2m_- + 1)K_{m_- +\frac{1}{2}}K_{m_- -\frac{1}{2}}\right]\left[K_{C+\frac{1}{2}}(1+\mu_2) + k\epsilon K_{C-\frac{1}{2}}\mu_3\right]\end{aligned}$$

$$f_5 = i\frac{K_{m_- +\frac{1}{2}}}{K_{m_- -\frac{1}{2}}} \tag{44}$$

with the denominator given by

$$D = i\left[K^2_{m_--\frac{1}{2}}K_{C+\frac{1}{2}}(1+\mu_2) + k\epsilon K^2_{m_--\frac{1}{2}}K_{C-\frac{1}{2}}\mu_3 - k\epsilon K_{m_-+\frac{1}{2}}K_{m_--\frac{1}{2}}K_{C+\frac{1}{2}}\mu_3\right] \tag{45}$$

In exactly the same way, we may also express $\psi^+_{i\epsilon}$ in terms of $\psi^-_{i\epsilon}$, with the result

$$\psi^+_{i\epsilon} = Q_{ij}\psi^-_{j\epsilon} \tag{46}$$

where

$$Q_{ij} = \overline{f}_1\gamma_i\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k}\gamma_j + \overline{f}_2\frac{k_i}{k}\gamma_j + \overline{f}_3\gamma_i\frac{k_j}{k} + \overline{f}_4\frac{k_i}{k}\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k}\frac{k_j}{k} + \overline{f}_5\frac{\mathbf{k}\cdot\boldsymbol{\gamma}}{k}\delta_{ij} \tag{47}$$

Conjugating (42) and (46) gives us the additional relations

$$\overline{\psi}^-_{i\epsilon} = \overline{\psi}^+_{j\epsilon}\overline{O}_{ij} \tag{48}$$

$$\overline{\psi}^+_{i\epsilon} = \overline{\psi}^-_{j\epsilon}\overline{Q}_{ij} \tag{49}$$

It is now possible to write (38) in a simple form. We will, in (56) below, consider a case in which it is necessary to express (38) in terms of only $\overline{\psi}^+$ and ψ^-. This is easily done by means of (46) and (48). We break up the field into + and − pieces

$$\begin{aligned} I &= \frac{1}{2}\int \mathrm{d}^d x\,\mathrm{d}^d y\sqrt{g}\int\frac{\mathrm{d}^d k}{(2\pi)^d}e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\Big(\overline{\psi}^+_{i\epsilon}(\mathbf{k})\psi^+_{i\epsilon}(-\mathbf{k}) + \overline{\psi}^-_{i\epsilon}(\mathbf{k})\psi^-_{i\epsilon}(-\mathbf{k}) \\ &\quad + \overline{\psi}^+_{i\epsilon}(\mathbf{k})\gamma_i\gamma_j\psi^+_{j\epsilon}(-\mathbf{k}) + \overline{\psi}^-_{i\epsilon}(\mathbf{k})\gamma_i\gamma_j\psi^-_{j\epsilon}(-\mathbf{k})\Big) \\ &= \frac{1}{2}\int \mathrm{d}^d x\,\mathrm{d}^d y\sqrt{g}\int\frac{\mathrm{d}^d k}{(2\pi)^d}e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\left(\overline{\psi}^+_{i\epsilon}(\mathbf{x})\Omega_{ij}(\mathbf{x}-\mathbf{y})\psi^-_{j\epsilon}(\mathbf{y})\right) \end{aligned} \tag{50}$$

and then write the correlator as

$$\Omega_{ij}(\mathbf{x}-\mathbf{y}) = \int\frac{\mathrm{d}^d k}{(2\pi)^d}e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\left[\gamma_i\gamma_l Q_{lj} + \overline{O}_{li}\gamma_l\gamma_j + Q_{ij} + \overline{O}_{ji}\right] \tag{51}$$

Since we wish to do the k-integral in (50), we must expand O and Q (and thus each f) and determine the power of both k and ϵ in each term. Firstly, the terms containing an even, non-negative power of k must be discounted, since they will vanish by the integral formula (39). Factors of the form k_i do not pose a problem here; such a factor can be converted to

$i\partial_i^{(x)}$, taking it outside the integral. Secondly, we must keep only leading-order terms in ϵ of what remains. To this end we introduce the notation

$$G_{\alpha\beta\gamma} \equiv K_{C+\alpha}(k\epsilon)K_{m_-+\beta}(k\epsilon)K_{m_-+\gamma}(k\epsilon) \tag{52}$$

and since we will always deal with $\alpha, \beta, \gamma = \pm 1/2$, we abbreviate this further in an obvious way. The first four f functions may now be written as

$$\begin{aligned}
f_1 &= \frac{1}{D}\big[C_1 G_{---} + C_2 G_{++-} + C_3 G_{-+-} k\epsilon + C_4 G_{+++} k\epsilon\big] \\
f_2 &= \frac{1}{D}\big[C_5 G_{++-} + C_6 G_{-+-} k\epsilon + C_7 G_{+--} k\epsilon \\
&\quad + C_8 G_{+++} k\epsilon + C_{10} G_{++-}(k\epsilon)^2 + C_{11} G_{---}(k\epsilon)^2\big] \\
f_3 &= \frac{1}{D}\big[C_{12} G_{-+-} k\epsilon + C_{13} G_{+++} k\epsilon + C_{14} G_{++-}\big] \\
f_4 &= \frac{1}{D}\big[C_{15} G_{+++} k\epsilon + C_{17} G_{++-}(k\epsilon)^2 + C_{18} G_{+--} k\epsilon \\
&\quad + C_{19} G_{++-} + C_{20} G_{---}(k\epsilon)^2 + C_{21} G_{-+-} k\epsilon\big]
\end{aligned} \tag{53}$$

where $C_{1\ldots21}$ are constants which can be read off from (44). Only the following will be needed

$$\begin{aligned}
C_1 = \frac{1+\mu_1}{d} \qquad\qquad C_2 = \frac{1+\mu_2}{d}\left(\frac{1+\mu_1}{m_1} - 1\right) \\
C_{18} = -C_5 = -C_{14} = \frac{(1+\mu_1)(1+\mu_2)}{m_1} \qquad\qquad C_{19} = (2m_- + 1)C_{18}
\end{aligned} \tag{54}$$

Now we use the small-argument expansion of the modified Bessel function

$$K_\nu(z) = \frac{1}{2}\Gamma(-\nu)\left(\frac{z}{2}\right)^{\nu}(1+\ldots) + \frac{1}{2}\Gamma(\nu)\left(\frac{z}{2}\right)^{-\nu}(1+\ldots) \tag{55}$$

where the dots indicate successive even powers of z. Clearly, which term we consider to be leading-order depends on the order of the Bessel function. To settle this point, from now on we turn our attention to the specific case

$$m_- > \frac{1}{2} \qquad\qquad C < -\frac{1}{2} \tag{56}$$

Other cases may be considered in a similar fashion. A quick inspection of the f's shows that in this case, it is ψ^- which may be specified on the boundary, so we will need to make use of (47) and (48), as mentioned above (50).

We apply (55) to obtain the leading-order term in the denominator $\overline{D}$,

$$\overline{D} \doteq \frac{i}{8} M \left(\frac{k\epsilon}{2}\right)^{C-2m_- - \frac{3}{2}} \tag{57}$$

where $M = -(1+\mu_1)\Gamma(1/2 - C)\Gamma(1/2 + m_-)^2$. Here we have introduced the dotted equal sign $\doteq$ which denotes equality up to leading order in ϵ, discounting terms which vanish when integrated due to their power of k, as explained above (52).[6]

Expanding the **k**-dependent part of a general term from $\overline{O}$ or Q, we find

$$\begin{aligned} \frac{\overline{G_{\alpha\beta\gamma}}(k\epsilon)^P}{\overline{D}k^l} \doteq\ & \frac{-i2^P}{M} \left[\Gamma(\alpha - C) \left(\frac{k\epsilon}{2}\right)^{\frac{1}{2}-\alpha+P} + \Gamma(C-\alpha) \left(\frac{k\epsilon}{2}\right)^{\frac{1}{2}-2C+\alpha+P} \right] \\ & \times \left[\Gamma(\beta - m_-) \left(\frac{k\epsilon}{2}\right)^{\frac{1}{2}+2m_- -\beta} + \Gamma(m_- - \beta) \left(\frac{k\epsilon}{2}\right)^{\frac{1}{2}+\beta} \right] \\ & \times \left[\Gamma(\gamma - m_-) \left(\frac{k\epsilon}{2}\right)^{\frac{1}{2}+2m_- -\gamma} + \Gamma(m_- - \gamma) \left(\frac{k\epsilon}{2}\right)^{\frac{1}{2}+\gamma} \right] \end{aligned} \tag{58}$$

In the expansion of this product, we will refer to individual terms by the signs of α, β, and γ in the exponent. By inspection it is seen that the $-\alpha+\beta+\gamma$ term is leading-order in ϵ; therefore we should ask whether this term will vanish when we do the **k**-integral. Table 1 shows on the LHS all instances, in (53), of the general term (58). It should be noted that there are also factors of k^{-1} which come from (43) and (47) so that, for example, we should consider $\frac{\overline{f_4}}{k^3}$ rather than just $\overline{f_4}$. On the RHS are the resulting powers of ϵ and k in the leading-order term in (58). We see that all elements in the table will vanish when we integrate over **k** except the two entries indicating k^{-2}. However, when this part of $\overline{f_4}$ is evaluated by substituting $\overline{C_{18}}$ and $\overline{C_{19}}$ from (54), we see that these two terms neatly cancel each other. Thus, the leading-order term in (58) will always vanish when we integrate. Since it cannot contribute, we must analyse the seven remaining higher-order terms to see which do. By similar arguments we see that the next-order terms in (58) (for generic $\alpha\beta\gamma$) are the $-\alpha-\beta+\gamma$, $-\alpha+\beta-\gamma$, and $+\alpha+\beta+\gamma$ terms. We will make no assumption as to whether C is larger or smaller in magnitude than m_-, so we must keep all three of these.[7] It is also assumed that the masses are not special in that when we integrate a term of the form $k^{(masses)}$, it does not vanish. Considering again each instance of the general term (58),

[6] In the case of (57), the $\doteq$ functions in only the first way since $\overline{D}$ is not integrated by itself.

[7] Actually, the $-\alpha+\beta-\gamma$ term turns out not to contribute anyway.

we find that the leading-order terms go as ϵ^{2m_-} and ϵ^{-2C}, and that they correspond only to the instances $\alpha\beta\gamma = ---$ and $\alpha\beta\gamma = ++-$, both with $P = 0$.[8] Hence, the only terms in (53) which will survive the $\epsilon \to 0$ limit and the **k**-integration are the $\overline{C_1}$, $\overline{C_2}$, $\overline{C_5}$, $\overline{C_{14}}$, and $\overline{C_{19}}$ terms,

$$\frac{\overline{f_1}}{k} \doteq \overline{C_1}\frac{\overline{G_{---}}}{\overline{D}k} + \overline{C_2}\frac{\overline{G_{++-}}}{\overline{D}k} \qquad \frac{\overline{f_2}}{k} \doteq \overline{C_5}\frac{\overline{G_{++-}}}{\overline{D}k} \qquad \frac{\overline{f_3}}{k} \doteq \overline{C_{14}}\frac{\overline{G_{++-}}}{\overline{D}k} \qquad \frac{\overline{f_4}}{k^3} \doteq \overline{C_{19}}\frac{\overline{G_{++-}}}{\overline{D}k^3} \tag{59}$$

Writing out explicitly from (58) the terms which will contribute, according to the above analysis, we have

$$\begin{aligned} \frac{\overline{G_{---}}}{\overline{D}k^l} &\doteq \frac{-i}{M}\Gamma\left(\frac{1}{2}+C\right)\Gamma\left(m_- - \frac{1}{2}\right)^2\left(\frac{\epsilon}{2}\right)^{-2C}k^{-2C-l} \\ \frac{\overline{G_{++-}}}{\overline{D}k} &\doteq \frac{-i}{M}\Gamma\left(\frac{1}{2}-C\right)\Gamma\left(\frac{1}{2}-m_-\right)\Gamma\left(\frac{1}{2}+m_-\right)\left(\frac{\epsilon}{2}\right)^{2m_-}k^{2m_- -1} \end{aligned} \tag{60}$$

[8]The value of l in (58) does not matter here since it affects only the power of k which appears, and not ϵ.

Table 1: Powers of k and ϵ in each instance of the leading-order term in (58)

l	$\alpha\beta\gamma$	P	Power of k	Power of ϵ
1	$---$	0	0	1
1	$++-$	0	0	1
1	$-+-$	1	2	3
1	$+++$	1	2	3
1	$+--$	1	0	1
1	$++-$	2	2	3
1	$---$	2	2	3
3	$+++$	1	0	3
3	$++-$	2	0	3
3	$+--$	1	-2	1
3	$++-$	0	-2	1
3	$---$	2	0	3
3	$-+-$	1	0	3

From (44), $\overline{f_5}$ is of order ϵ^{2m_-}, and

$$\frac{\overline{f_5}}{k} \doteq -i\frac{\Gamma(\frac{1}{2}-m_-)}{\Gamma(\frac{1}{2}+m_-)}\left(\frac{\epsilon}{2}\right)^{2m_-} k^{2m_- -1} \tag{61}$$

Now the formula (39) can be used to find $\int \mathrm{d}k$ of (60). Substituting the results into (43) (conjugated) yields

$$\begin{aligned}
&\int \frac{\mathrm{d}^d k}{(2\pi)^d} e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\overline{O}_{ij} \doteq \\
&\frac{1}{M}\Bigg\{\frac{\overline{C_1}}{2\pi^{d/2}}\Gamma\left(m_- - \frac{1}{2}\right)^2\Gamma\left(\frac{d-2C-1}{2}\right)\gamma_j\boldsymbol{\gamma}\cdot\boldsymbol{\partial}\gamma_i\frac{\epsilon^{-2C}}{|\mathbf{x}-\mathbf{y}|^{d-2C-1}} \\
&-\frac{\overline{C_{19}}}{8\pi^{d/2}\left(\frac{1}{2}-m_-\right)}\Gamma\left(\frac{1}{2}-C\right)\Gamma\left(\frac{1}{2}+m_-\right)\Gamma\left(\frac{d+2m_- -3}{2}\right)\boldsymbol{\gamma}\cdot\boldsymbol{\partial}\partial_i\partial_j\frac{\epsilon^{2m_-}}{|\mathbf{x}-\mathbf{y}|^{d+2m_- -3}} \\
&+\frac{\Gamma\left(\frac{1}{2}-C\right)}{2\pi^{d/2}}\Gamma\left(\frac{d+2m_- -1}{2}\right)\left[\overline{C_2}\gamma_j\boldsymbol{\gamma}\cdot\boldsymbol{\partial}\gamma_i+\overline{C_5}\partial_i\gamma_j+\overline{C_{14}}\gamma_i\partial_j\right]\frac{\epsilon^{2m_-}}{|\mathbf{x}-\mathbf{y}|^{d+2m_- -1}} \\
&+\frac{\Gamma\left(\frac{d+2m_- -1}{2}\right)}{2\pi^{d/2}\Gamma\left(\frac{1}{2}+m_-\right)}\boldsymbol{\gamma}\cdot\boldsymbol{\partial}\frac{\epsilon^{2m_-}}{|\mathbf{x}-\mathbf{y}|^{d+2m_- -1}}\Bigg\}
\end{aligned} \tag{62}$$

Doing the derivatives and simplifying, we obtain

$$\begin{aligned}
\int \frac{\mathrm{d}^d k}{(2\pi)^d} e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\overline{O}_{ij} &\doteq +\frac{1}{\pi^{d/2}}\left(\frac{1+\mu_2}{m_1}+1\right)\frac{\Gamma\left(\frac{d+2m_- +1}{2}\right)}{\Gamma\left(\frac{1}{2}+m_-\right)} \\
\times\Bigg\{\boldsymbol{\gamma}\cdot|\mathbf{x}-\mathbf{y}|&\left(\delta_{ij}-2\frac{(x-y)_i(x-y)_j}{|\mathbf{x}-\mathbf{y}|^2}\right)+\frac{\gamma_j\boldsymbol{\gamma}\cdot|\mathbf{x}-\mathbf{y}|\gamma_i}{d}\Bigg\}\frac{\epsilon^{2m_-}}{|\mathbf{x}-\mathbf{y}|^{d+2m_- +1}} \\
&+\frac{1}{(1+\mu_1)\left(m_- - \frac{1}{2}\right)^2\pi^{d/2}}\frac{\Gamma\left(\frac{d-2C+1}{2}\right)}{\Gamma\left(\frac{1}{2}-C\right)}\gamma_j\boldsymbol{\gamma}\cdot|\mathbf{x}-\mathbf{y}|\gamma_i\frac{\epsilon^{-2C}}{|\mathbf{x}-\mathbf{y}|^{d-2C+1}}
\end{aligned} \tag{63}$$

Comparing (47) with (43), we see that it is unnecessary to calculate $\int \mathrm{d}k Q_{ij}$ separately; it can be obtained trivially from (63) by switching the i and j indices in the $\gamma_j\boldsymbol{\gamma}\cdot|\mathbf{x}-\mathbf{y}|\gamma_i$ terms.

The correlator $\Omega_{ij}(\mathbf{x}-\mathbf{y})$ from (51) may now be written

$$\begin{aligned}
\Omega_{ij}(\mathbf{x}-\mathbf{y}) = M_I&\left[\frac{\gamma_i\boldsymbol{\gamma}\cdot|\mathbf{x}-\mathbf{y}|\gamma_j}{d}+\boldsymbol{\gamma}\cdot|\mathbf{x}-\mathbf{y}|\left(\delta_{ij}-2\frac{(x-y)_i(x-y)_j}{|\mathbf{x}-\mathbf{y}|^2}\right)\right]\frac{\epsilon^{2m_-}}{|\mathbf{x}-\mathbf{y}|^{d+2m_- +1}} \\
&+M_{II}\gamma_i\boldsymbol{\gamma}\cdot|\mathbf{x}-\mathbf{y}|\gamma_j\frac{\epsilon^{-2C}}{|\mathbf{x}-\mathbf{y}|^{d-2C+1}}
\end{aligned} \tag{64}$$

Where M_I and M_{II} are constants. The two correlators contained in this expression can be separated by decomposing the boundary field $\psi_{i\epsilon}$ into two parts, projecting out the component orthogonal to γ_i. To use the AdS/CFT correspondence (1), we define the boundary fields[9]

$$\chi_{(0)} \equiv \gamma_j \psi_j \qquad \text{and} \qquad \psi_{i(0)} \equiv \psi_i - \frac{\gamma_i}{d}\chi_{(0)}$$

so that $\gamma_i \psi_{i(0)} = 0$.

Rewriting in terms of these new fields, while absorbing appropriate powers of ϵ, gives us two correlators, one for the conformal operator coupling to each boundary field

$$\langle \mathcal{O}_{i\alpha} \overline{\mathcal{O}}_{j\beta} \rangle = M_I \boldsymbol{\gamma}_{\alpha\beta} \cdot |\mathbf{x}-\mathbf{y}| \left[\delta_{ij} - 2\frac{(x-y)_i (x-y)_j}{|\mathbf{x}-\mathbf{y}|^2} \right] \frac{1}{|\mathbf{x}-\mathbf{y}|^{d+2m_-+1}} \tag{65}$$

$$\langle \mathcal{O}'_\alpha \overline{\mathcal{O}'}_\beta \rangle = M_{II} \boldsymbol{\gamma}_{\alpha\beta} \cdot |\mathbf{x}-\mathbf{y}| \frac{1}{|\mathbf{x}-\mathbf{y}|^{d-2C+1}} \tag{66}$$

and the scaling dimensions of the operators are $\Delta_{\mathcal{O}_i} = \frac{d}{2} + m_-$ and $\Delta_{\mathcal{O}'} = \frac{d}{2} - C$.

Each of these correlators is of the form required by conformal invariance [22]. As remarked below equation (14), the $\gamma^\mu \psi_\mu = 0$ case has been considered and yields only the first correlator in (65). It is interesting to note that since $\Delta_{\mathcal{O}'}$ depends on C, it is non-analytic in m_1, due to (14). Hence, the massless case cannot be recovered as the limit $m_1 \to 0$.

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Appendix

The following are useful identities involving the constants defined in (24) and (14).

$$\mu_1 = \frac{1-d-2m_2}{d-1-2m_-} \qquad \mu_2 = \frac{1-d+2m_2}{d-1+2m_-} \qquad \mu_3 = \frac{4m_1(1-d)}{d(d-1-2m_-)(d-1+2m_-)}$$

[9] The presence of the spinor field χ accounts for the correct number of degrees of freedom coming from the original $d+1$-dimensional Rarita-Schwinger field.

$$
\begin{aligned}
\frac{\mu_3}{d-1} &= \frac{1+\mu_1}{m_1}\frac{1+\mu_2}{d} \qquad\qquad \frac{1+\mu_1}{1+\mu_2} = 2m_- \frac{1+\mu_1}{m_1} - 1 \\
\frac{\mu_1 + \frac{m_+}{d-1} + \frac{1}{2}}{C+m_-} &= \frac{-2m_1}{d(d-1-2m_-)} = \frac{1+\mu_1}{d} \\
\frac{\mu_2 - \frac{m_+}{d-1} + \frac{1}{2}}{C+m_-} &= \frac{-2m_1}{d(d-1+2m_-)} = -\frac{1+\mu_2}{d}
\end{aligned}
$$

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